Seepage Characteristics Of Bingham Fluid In Pores Medium

Zheng jingyun[1], Xiao Sanxia[2], Jiang Guoping[3]
[1]Zhejiang Reclaim Construction Group Co., Ltd., Ningbo 31504, China
[2,3]School of Engineering, Fujian Jiangxia University, Fuzhou, Fujian, China

Abstract

The permeability properties for Bingham fluid is studied in this paper. Based on the fractal character of pore size distribution and tortuosity of capillaries, a physical conceptual model for permeability properties for Bingham fluid in pores medium is derived. The fractal expressions of flow velocity, flow rate and effective permeability for Bingham fluid flow in porous media have been presented. There are so many parameters such as the porosity, size of pores, tortuosity of capillaries are all considered in this study.

Keywords: unsaturated rocks, capillary pressure, Bingham fluid, model

1 Introduction

The knowledge of permeability properties for Bingham fluid is important to study the water and solute movement in pores medium. Permeability to water flow in pores medium is a key parameter in so many fields [1−8] as well as reservoir engineering, soil science and so on. The permeability properties for Bingham fluid is relative of the microstructure of the pores medium, and Bingham fluid is Non-Newton fluid which lead it very difficult to obtain the relative permeability of Bingham fluid in pores medium. There are so many parameters such as the porosity, size of pores, tortuosity of capillaries are all important to study the permeability properties for Bingham fluid. Fortunately, fractal theory has developed to investigate this problem with nonlinear science. As the early time, Sierpinski carpet is often used to study the the microstructure of pores medium. Luis [9] present a conceptual constitutive model to the fractal dimension to the parameters of the Brooks–Corey constitutive model. A Sierpinski space is as well as used to determine the spatial distribution for drainage network with the Gardon basin, France [10]. Permeability to water flow in unsaturated fractured rocks is a key parameter to be considered in many fields [1−8] such as soil science,, reservoir engineering, , and chemical engineering. Since the microstructures of real unsaturated fractured rocks are usually disordered and extremely complicated, which makes it very difficult to analytically find the relative permeability of the unsaturated fractured rocks. Many parameters such as size of pores, tortuosity of capillaries and the porosity are very important for water flow in unsaturated fractured rocks. These parameters are closely related to the geometric architecture of unsaturated fractured rocks. With the Sierpinski carpet ,the conceptual constitutive model presented by Luis [9] had used the fractal dimension to the parameters of the Brooks–Corey constitutive model. A Sierpinski space was also adopted to characterize the spatial distribution of a drainage network in the Gardon basin, France [10].When use the Sierpinski carpet, the tortuosity fractal dimension D_T is often neglected in the investigation which is a important parameter for unsaturated fractured rocks . But Sierpinski carpet can't be determine the tortuosity fractal dimension. So the tortuosity fractal dimension is often neglected in the past investigation, In this paper, fractal theory is used to study the permeability of Bingham fluid flow in pores medium. The tortuosity fractal dimension is also considered in this study.

2 The model

The study model is presented in model 1.The model is Composed of many capillary. In the model, tortuosity fractal dimension is considered.

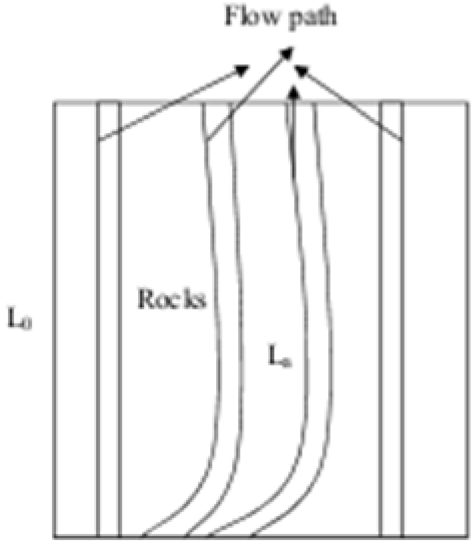

Figure.1 The model presented

The Constitutive model for Non-newtonian Bingham fluid fluid is

$$\tau = \tau_0 + \mu \dot{\gamma} \qquad (1)$$

The tensor τ is called the deviator or the deviatoric stress, When $\tau \prec \tau_0$ where τ is the yield stress. When $\tau \succ \tau_0$, where μ is the (limiting) fluid viscosity. $\dot{\gamma}$ is Shear rate

Fluid velocity can be obtained by the following formula

$$v = {}^{r}\!/_{\tau_r} \int_0^{\tau_r} \dot{\gamma}\, d\tau \qquad (2)$$

Where v is the fluid velocity.

From Equations (1) and (2),we get,

$$\tau_r = \frac{4\mu v}{r} + \frac{4}{3}\tau_0 \qquad (3)$$

Where r is the radius of tortuosity.

The relationship between fluid pressure and shear force are presented as follows,

$$\Delta p \pi r^2 = 2\pi \tau_r r \Delta L_a \qquad (4)$$

Where Δp is the pressure difference.

3 The fractal theory

The pore microstructures (both pore-interfaces the and the pore sizes) of unsaturated fractured rocks exhibit the fractal characteristics[11], It has been proven that the cumulative size-distribution of pores whose sizes are equal to or greater than the size λ follows the fractal scaling law[12-13].When Bingham fluid flow through the pores medium, the capillaries may be tortuous. These tortuous capillaries may be expressed by fractal equation [8]

$$L_a(r) = L_0^{D_T} r^{1-D_T} \tag{5}$$

where D_T is the tortuosity fractal dimension, whose range is from 1 to 2.
From Equations (3),(4) and (5),we get,

$$\tau_r = \frac{\Delta p}{2r\Delta L_a} = \frac{\Delta p}{2L_0^{D_T} r^{2-D_T}} \tag{6}$$

$$v = \frac{r}{8\mu}\left(\frac{\Delta p}{L_0^{D_T} r^{2-D_T}} - \frac{8}{3}\tau_0\right) \tag{7}$$

$$q = v \times \pi r^2 = \frac{\pi r^3}{8\mu}\left(\frac{\Delta p}{L_0^{D_T} r^{2-D_T}} - \frac{8}{3}\tau_0\right) \tag{8}$$

Where q is the flow rate of a single tortuosity.
The pore microstructures (pore-interfaces, the pore sizes) of pores medium have the fractal characteristics[11], It has been proven that the cumulative size-distribution of pores whose sizes are equal to or greater than the size λ follows the fractal scaling law [12-13].

$$N(L \ge \lambda) = \left(\frac{r_{\max}}{r}\right)^{D_f} \tag{9}$$

where D_f is the fractal dimension of pores, λ is the diameter, L is the length scale, N is the total number of pores whose sizes are greater than and equal to the diameter r .The derivative of Eq. (5) respected to r can be written as

$$-dN = D_f r_{\max}^{D_f} r^{-(D_f+1)} dr \tag{10}$$

We can get,

$$Q = \int_{r_{Min}}^{r_{Max}} q\, dN = \int_{r_{Min}}^{r_{Max}} \frac{\pi r^3}{8\mu}\left(\frac{\Delta p}{L_0^{D_T} r^{2-D_T}} - \frac{8}{3}\tau_0\right) dN \tag{11}$$

With Eq.(10) and (11), we get,

$$Q = \int_{r_{Min}}^{r_{Max}} -\frac{\pi r^3}{8\mu}\left(\frac{\Delta p}{L_0^{D_T} r^{2-D_T}} - \frac{8}{3}\tau_0\right) D_f r_{\max}^{D_f} r^{-(D_f+1)} dr \tag{12}$$

$$Q = -\frac{\Delta p \pi D_f r_{\max}{}^{D_f}}{8 L_0{}^{D_T} \mu} \int_{r_{Min}}^{r_{Max}} r^{D_T - D_f}\, dr + \frac{\pi}{3\mu} \tau_0 D_f r_{\max}{}^{D_f} \int_{f\; r_{Min}}^{r_{Max}} r^{2-D_f}\, dr \qquad (13)$$

With Eq.(13) we get,

$$Q = -\frac{\Delta p \pi D_f r_{\max}{}^{D_f}}{8 L_0{}^{D_T} \mu (1 + D_T - D_f)} (r_{\max}{}^{1+D_T-D_f} - r_{\min}{}^{1+D_T-D_f})$$
$$+ \frac{\pi}{3(3-D_f)\mu} \tau_0 D_f r_{\max}{}^{D_f} (r_{\max}{}^{3-D_f} - r_{\min}{}^{3-D_f}) \qquad (14)$$

Where Q is the flow rate of the Non-newtonian Bingham fluid.

4 Results and discussion

Based on the fractal theory, the fractal expressions of flow velocity ,flow rate and effective permeability for Bingham fluid flow in porous media have been presented. The presented fractal model is a function of fluid characteristic parameters as well as pressure drop and structural parameters of porous media. The presented model has clear physical meaning and relative properties of Bingham fluid flow with the structural parameters in porous media, which help to understand flow mechanism for non-Newtonian fluid through porous media.

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